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A FUZZY MODEL OF THE PROBLEM OF OPTIMAL PLACEMENT AND INTEGRATION OF OFFSHORE OIL AND GAS PLATFORMS

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Abstract. Platforms need to be constructed on the sea to drill directional oil and gas wells. All these processes have great importance in economy base. Therefore, there is significant need for minimizing the cost of the processes. It is known that when planning directional sea oil and gas-wells, platforms are set up in certain places, and certain number of directional wells are drilled from these platforms. Each well can only be drilled from the platform and it must be known hot to bond platforms with each other. In this paper, the problem of optimum placement and combination of the platforms with each other is studied, related mathematical models is analyzed and a new model is proposed. The model is stated as specially constrained 0-1 linear programming problem. Also in the paper, taking some parameters as fuzzy information on certain conditions a new mathematical model is developed. If fuzzy parameters are used adaptation to the actual condition will be good enough and this will make it easier to decide better with using subjective expert knowledge and experience.

Keywords: oil and gas platforms, optimal placement and connection of the platforms, fuzzy model, linear 0-1 programming, directional well, Minimum Spanning Tree, Simple Location Problem, communication lines

AMS Subject Classification: 90B80, 90C09, 90C27, 90C70, 90C90.

1. Introduction

Nowadays vast majority of onshore oil and gas reserves are drained [1]. That is why offshore reserves have a huge importance in terms of economy. The number of the wells to be drilled in order to reach to the oil and gas reserves with a sloping route are determined by geology experts**.**

First of all platforms need to be constructed for drilling, and after that communication and transportation lines must be boned which needs sizeable amount of money [2]. Therefore, modelling problems that contains optimal execution of all these process, developing efficient algorithms to solve these problems and designing software systems for these algorithms have great importance [3].

Some of the costs for this problem are not exact. For example, every platform has not same standards, they varies according to the height, capacity and etc. In addition, the cost of the platforms may vary depending on the platform floor, establishing communication between platforms and transportation lines.

New wells may be needed in time. Therefore, fuzzy constraints and fuzzy criteria are involved in this problem [4].

If certain parameters (greatest value for the angle of drilling, drilling expenditure, integration of the platforms etc.) are given as fuzzy constraints, then performance of the prepared system of the program will be higher, adaptation to the actual condition will be good enough and this will make it easier to decide better with using subjective expert knowledge and experience [5]. Thus, it may provide decision-maker to give a more realistic decisions than the subjective expert knowledge and the experience.

Mathematical model with exact data for optimal placement of platforms that is drilled directionally given in [6] and solution algorithm for the model is suggested in [7]. In [8] an interactive algorithm for the problem is given for which some parameters are given as fuzzy information.

While drilling directionally, determining optimal profile for the well is another optimization problem [9]. Also, a mathematical model for the problem of optimal integration and combination of oil-and gas platforms in sea is given in [10].

In our paper, we analyze related mathematical models for the problem and take certain parameters as fuzzy information to represent a new model for the optimal placement and integration of oil-and gas platforms which are drilled directionally.

2. A mathematical formulation of the problem

It is known that when planning directional offshore oil and gas-wells, platforms are set up in certain places, and certain number of directional wells are drilled from these platforms (Figure 1).

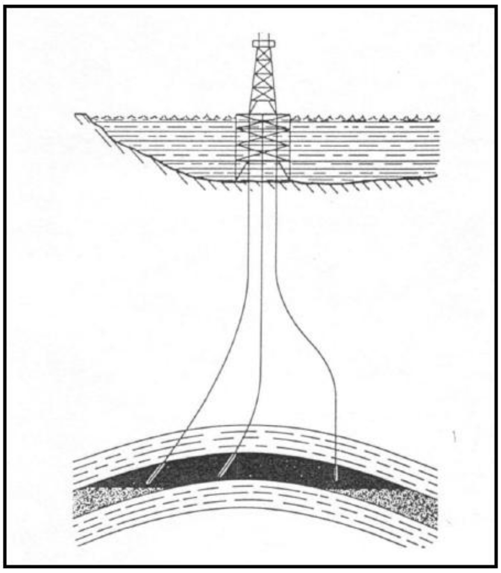

Figure 1. Directional drilling from an offshore platform.

Here, each well can only be drilled from the platform. In this case, the aim of the problem is to reduce the communication and transportation costs from one platform to the other platforms and to the land (or a platform which is already constructed). Thats why answers to the following questions must be found:

- In which parts of the sea platforms should be constructed in order to minimize cost?
- Which well should be drilled from which platforms?
- How should be combination of the platforms with each other and with ground (or a platform which is already installed)?

At first sight it can be considered to reduce the number of the platforms in order to get the minimum cost. However, it is not true for every condition. Since reducing the number of platforms may end up with increasing distance between each platforms (and possible new platforms), cost of transportation and communication lines will scale up. So the aim is not only to reduce the cost of the platform, but to reduce the total cost along with communication and transportation lines. In [10] this problem is discussed and a mathematical model was given considering some special cases (Figure 2).

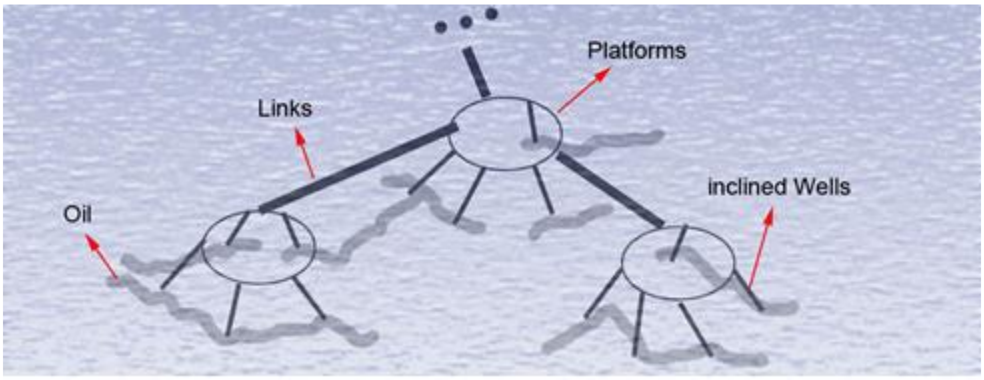

Figure 2. Example visual for the integration of the platforms.

Suppose that *n* is the number of the wells to be drilled in this problem. Lets show this wells with j $\left(j=\overline{1,n}\right)$. In order to establish off shore platforms, assume that *m* possible centers which are selected and lets show them with i $\left(i=\overline{1,m}\right)$. Naturally, not all of the platforms will be installed in all these centers. Lets show the drilling cost of *j* th well from *i* th platform *i* with c_{ij} $\left(i=\overline{1,m};\ j=\overline{1,n}\right)$. If depth of the area and the angle that platform occupies is given, then we can calculate the cost c_{ij} with known methods. Lets show the cost for installation of platform in *i* th center with a_i. This cost consists of two parts which does not depend on the number of the wells which will be drilled:

$$a_i = a_i^{'} + a_i^{''}$$

Here, $a_i^{'}$- is the cost of the installation of the platform, $a_i^{''}$- is the cost for full installation of the platform. In that case, if *i* th platform will be installed, cost of

$a_i \ \left(i=\overline{1,m}\right)$ will also be added to the total cost of wells to be drilled from this platform.

Each platform has a certain capacity. So, lets show the maximum number of wells to be drilled from the *i* th platform with $p_i \ \left(i=\overline{1,m}\right)$.

Let's determine the following variables:

$$x_{ij} = \begin{cases} 1, & \text{if } j \text{ th well is drilled from platform } i \\ 0, & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{if platform } i \text{ is constructed} \\ 0, & \text{otherwise} \end{cases}$$

So, then, cost to drill the wells from platforms will be

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

and cost to install platform will be

$$\sum_{i=1}^{m} a_i y_i$$

Each well must be drilled only from one platform. We will guarantee it with following equations:

$$\sum_{j=1}^{m} x_{ij} = 1, \ \left(j=\overline{1,m}\right)$$

In order to ensure $p_i \ \left(i=\overline{1,m}\right)$ amount of well to be drilled in *i* th platform, equations given below should be provided:

$$\sum_{j=1}^{n} x_{ij} \le p_i \cdot y_i \ , \quad \left(i=\overline{1,m}\right)$$

$x_i = \sum_{j=1}^{n} x_{ij} \ \left(i=\overline{1,m}\right)$ is the number of the wells to be drilled in *i* th platform, then

$$c_i = f(x_i) = a_i + k_i x_i \, .$$

Here, k_i is the cost when 1 is added to the number of the wells which will be drilled from *i* th platform.

Another cost is the integration cost. Let's determine the following variable for it:

$$z_{ij} = \begin{cases} 1, & \text{if } j \text{ th platform is connected with platform } i \\ 0, & \text{otherwise} \end{cases}$$

Let's l_{ij} be cost to integrate the *i* th platform with *j* th platform (total cost of communication and transportation). Then, cost for combination of the platforms with each other will be

$$\sum_{i=1}^{m}\sum_{k=1}^{m} l_{ik} z_{ik}$$

It is obvious that, in order to integrate two platforms both of the platforms must be installed.

$$z_{ik} \le y_i \cdot y_k \, , \ \left(i, k = \overline{1, m}\right).$$

On the other hand, platforms must be integrated with a land (or to the platform which is installed before) through a platform. If we symbolically mark shore as 0 th platform, it should hold:

$$\sum_{i=1}^{m} z_{oi} = 1 .$$

Furthermore, each platform must be combined with minimum another platform.

$$\sum_{i=1}^{m} z_{ik} \ge y_k \, , \ \left(k = \overline{1, m}\right)$$

Thus, a mathematical model of this problem will be like following:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} a_i y_i + \sum_{i=1}^{m}\sum_{k=1}^{m} l_{ik} z_{ik} + \sum_{i=1}^{k} l_{0i} z_{0i} \to \min \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} = 1, \ \left(j = \overline{1, n}\right) \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} \le p_i \cdot y_i \, , \ \left(i = \overline{1, m}\right) \tag{3}$$

$$z_{ik} \le y_i \cdot y_k \, , \ \left(i, k = \overline{1, m}\right) \tag{4}$$

$$\sum_{i=1}^{m} z_{oi} = 1 \tag{5}$$

$$\sum_{i=1}^{m} z_{ik} \ge y_k \, , \ \left(k = \overline{1, m}\right) \tag{6}$$

$$x_{ij} = 1 \vee 0, \ \ y_i = 1 \vee 0, \ \ z_{ik} = 1 \vee 0, \ \ \left(i, k = \overline{1, m}, \, j = \overline{1, n}\right) \tag{7}$$

The problem (1)-(7) is a 0-1 nonlinear integer programming problem. But, if we will change the condition (4) as $z_{ik} \le \frac{1}{2}\left(y_i + y_k\right)$, $(i, k = \overline{1, m})$ then the model will be linear.

In the model (1)-(7) the way of the integration of the platforms is considered. Taking some parameters as fuzzy information and also considering the way of integration, new mathematical model is given in the section below.

3. A mathematical model based on fuzzy information

We would like to state that, below features must be considered in order to make the model become more suitable to real situation:

1. It is suggested that, angle in directional drilling should be fuzzy data, because it makes the model flexible and allow it to investigate different situations.
2. Constructing platforms and drilling wells have inherently fuzzy characteristics. Because this process can change due to depth of the sea, tectonic movement, content of the layers, oil and gas reserves in the layers. Therefore, making mathematical model with fuzzy parameters will fit for the purpose.
3. If fuzzy parameters are used adaptation to the actual condition will be good enough and this will make it easier to decide better with using subjective expert knowledge and experience.

Let's explain the details of the fuzzy mathematical model. Let XOY shows sea level, and positive side of the Z line face to the bottom of the sea. Then, head of the each well j will be the coordinates (x_j, y_j, z_j) $j=\overline{1,n}$. We can assume that coordinates are not negative. So, $x_j \geq 0,\ \ y_j \geq 0,\ \ z_j \geq 0,\ \ j=\overline{1,n}$. By changing origin of coordinates, we can satisfy these constraints whenever it is needed.

Suppose that, coordinates of the places where platforms will be settled are $(u_i, v_i, 0)$ $i=\overline{1,m}$. Then, values c_{ij} $\left(i=\overline{1,m},\quad j=\overline{1,n}\right)$ for every i and j will be calculated by using coordinates (x_j, y_j, z_j) and $(u_i, v_i, 0)$.

Let's suppose that we have k types of platform by their capacity. A capacity of platform shows the maximum number of wells that can be drilled from this platform. Of course, if platform will be established or not for each center i is the object of interest.

For the following process, considering requirement for well and platform, perspective factors such as repairing and maintenance and other opportunities to installing or not installing platform in every i th center or identifying its capacity can be determined by different importance degree. One of the "Perspective Factors", for example, can be the drilling new wells from the platform. Obviously, the evaluation of this condition in advance is based only intuition and experience of the experts. Taking these situations into account, in model (1) – (5) it is more appropriate to declare $p_i \quad (i=\overline{1,m})$ values as fuzzy. Therefore, in equation (3) we can accept $p=\max\limits_{i=\overline{1,m}} p_i$ instead of value $p_i \quad (i=\overline{1,m})$. Thereby in this case the set of possible solutions will expand and as a result we can get more general solution. After finding such solution, in second part taking into account the fuzzy features of $p_i \quad (i=\overline{1,m})$, equation (3) will be concretized for each platform.

Since we have the angle gap for every platform, it should hold

$$|M_i, N_j| x_{ij} \leq r,\ \left(i=\overline{1,m}, j=\overline{1,n}\right).$$

Here $|M_i, N_j|$ is the distance between the point - M_i (with the center where *i* th platform is placed) and N_j (projection point where the top of the *j* th well is placed), r is the angle, which well is allowed to drill, can be calculated with a formula given below:

$r = h \cdot tg\,\beta_{\max}$, here *h* is the depth of the sea, $\beta_{\max}$ is the possible maximum drilling angle (Figure 3)

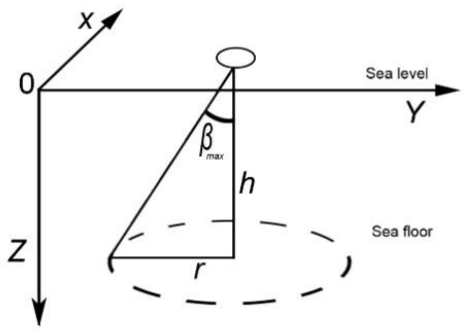

Figure 3. Location of the coordinate axes and demonstration of drilling angle of the platforms.

Thus, a mathematical model based on fuzzy criteria and constraints of the problem is as following:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} + \sum_{i=1}^{m}\tilde{a}_i y_i + \sum_{i=1}^{m}\sum_{i=1}^{m}\tilde{l}_{ik}z_{ik} + \sum_{i=1}^{k}\tilde{l}_{0i}z_{0i} \rightarrow \widetilde{\min} \quad (8)$$

$$\sum_{i=1}^{m} x_{ij} = 1, \ \left(j = \overline{1,n}\right) \quad (9)$$

$$\sum_{j=1}^{n} x_{ij} \le \tilde{p}_i \cdot y_i, \quad \left(i = \overline{1,m}\right) \quad (10)$$

$$z_{ik} \le \frac{1}{2}\left(y_i + y_k\right), \ \left(i,k = \overline{1,m}\right) \quad (11)$$

$$\sum_{i=1}^{m} z_{oi} = 1 \quad (12)$$

$$\sum_{i=1}^{m} z_{ik} \ge y_k, \ \left(k = \overline{1,m}\right) \quad (13)$$

$$|M_i, N_j| x_{ij} \le \tilde{r}, \ \left(i = \overline{1,m},\ j = \overline{1,n}\right) \quad (14)$$

$$x_{ij} = 1 \vee 0, \ y_i = 1 \vee 0, \ z_{ik} = 1 \vee 0, \ z_{0k} = 1 \vee 0, \ \left(i,k = \overline{1,m},\ j = \overline{1,n}\right) \quad (15)$$

Here $\tilde{c}_{ij}, \left(i = \overline{1,m},\ j = \overline{1,n}\right)$ - is a fuzzy number which shows the cost to drill *j* th well from *i* th platform. This parameter varies depending on the depth of the sea, durability of the seabed and tectonic situation which is confidential information because of the company policies.

$\tilde{a}_i$, $\left(i=\overline{1,m}\right)$ - is a fuzzy number which shows the cost needed to construct platform in *i* th center. This parameter changes between 200-900 million dollars. Due to developing technology, platforms can be renewed and cost can change.

$\tilde{p}_i$ - is the number which limits the capacity of the platforms. About 50 wells can be drilled from modern platforms.

$\tilde{r}$ - is the fuzzy number (radius) which determines the maximum slope angle. Drilling angle varies between the 15-75 degrees. Due to collapse of the layers drilling angle can be reduced.

$\tilde{l}_{ij}$ - is the fuzzy number which shows the cost of the communication and transportation to connect the *i* th platform with *j* th platform. This value may vary in huge ranges because there is not any limit between the platforms.

$\tilde{l}_{0t}$ - is the fuzzy number which shows the cost to make a connection between land or 0 th and *t* th platforms which were exist before.

We would like to state that, $"\underset{\sim}{<}"$ link and $\widetilde{\min}$ condition operations on the fuzzy information on the problems (8) – (15) are determined by the proper fuzzy set operations [4].

Both in model (8)-(15) and (1) – (7) conditions have the identical meaning. As it is seen, problem (8)-(15) is the 0-1 Programming Problem with fuzzy coefficient [11]. This problem, is the union of the “Minimum Spanning Tree” and “Simple Location” problems, so convenient algorithms can be used to solve the problem [12]. We can use the methods in [13]-[15] to solve problem.

3. Conclusion

In our paper, we analyze related mathematical models for the problem and take some parameters and constraints with fuzzy information to formulate a new model for the optimal placement and integration of oil-and gas platforms which is drilled directionally. If fuzzy parameters are used, adaptation to the actual condition will be good enough and this will make it easier to decide better with using subjective expert knowledge and experience.

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